Chapter 5 - Day 3

Recall the Chain Rule

$$(\square^n)' = n\,\square^{n-1} \cdot \square'$$

__Ex__: Suppose $F(x) = g(h(x))$
if $h(3)=2$, $h'(3)=1$, $g(3)=5$, $g'(3)=7$,
$g(2)=1$, $g'(2)=3$, find $F'(3)$.

$$F'(x) = g'(h(x)) \cdot h'(x)$$

$$F'(3) = g'(h(3)) \cdot h'(3)$$

$$= g'(2) \cdot h'(3)$$

$$= 3 \cdot 1 = \boxed{3}$$

Ex: let $g(x) = f(x^2+3(x-1)+5)$ and $f'(6) = 21$. find $g'(1)$.

by chain rule

$$g'(x) = \left[f'(x^2+3(x-1)+5)\right] \cdot (2x+3)$$

$$g'(1) = f'(1+0+5) \cdot (2(1)+3)$$

$$= f'(6) \cdot 5$$

$$= 21 \cdot 5 = \boxed{105}$$

Ex: Suppose $h(x)=\sqrt{f(x)}$ and the equation of the tangent line to $f(x)$ at $x=-1$ is $y=9+3(x+1)$.

Find $h'(-1)$.

$h(x)=(f(x))^{1/2}$

chain rule $h'(x)=\frac{1}{2}(f(x))^{-1/2}\cdot f'(x)$

$h'(-1)=\frac{1}{2}(f(-1))^{-1/2}\cdot f'(-1)$

* note: we need $f(-1)$ and $f'(-1)$.

* use the tangent line *

tangent line $y-9=3(x+1)$

Slope $m=3 \rightarrow f'(-1)=3$

Point $(-1,9) \rightarrow f(-1)=9$

thus $h'(-1)=\frac{1}{2}(f(-1))^{-1/2}\cdot f'(-1)$

$=\frac{1}{2}(9)^{-1/2}\cdot 3=\frac{1}{2\sqrt{9}}\cdot 3=\frac{3}{2\cdot 3}=\boxed{\frac{1}{2}}$

Higher Derivatives

the second derivative

$$y'' = f''(x) = (f'(x))'$$

$$\frac{d^2}{dx^2}(f(x)) \quad \text{or} \quad \frac{d^2y}{dx^2}$$

the third derivative

$$f'''(x) = (f''(x))'$$

for larger derivatives, we change notation,

$$f^{(4)}(x), f^{(5)}(x), \ldots$$

Ex: let $f(x) = x^4 - 3x^3 + 5x^2 + 2x + 1$

find $f''(x)$.

$f'(x) = 4x^3 - 9x^2 + 10x + 2$

$f''(x) = (f'(x))' = \boxed{12x^2 - 18x + 10}$

Ex: $g(x)=\dfrac{3x+5}{2x+1}$ find $\dfrac{d^2g}{dx^2}$

$$\frac{dg}{dx}=\frac{(3x+5)'(2x+1)-(3x+5)(2x+1)'}{(2x+1)^2}$$

$$=\frac{3(2x+1)-(3x+5)(2)}{(2x+1)^2}$$

$$=\frac{6x+3-6x-10}{(2x+1)^2}=\frac{-7}{(2x+1)^2}$$

$$\frac{d^2g}{dx^2}=\frac{(-7)'(2x+1)^2-(-7)\left((2x+1)^2\right)'}{\left((2x+1)^2\right)^2}$$

$$=\frac{0(2x+1)^2+7\left(2(2x+1)(2)\right)}{(2x+1)^4}$$

$$=\frac{28(2x+1)}{(2x+1)^4}=\frac{28}{(2x+1)^3}$$

Ex: let $h(x) = \sqrt{x}$. find $h^{(3)}(x)$.

$h(x) = x^{1/2}$

$h'(x) = \frac{1}{2} x^{-1/2}$

$h''(x) = \frac{1}{2}\left(\frac{-1}{2}\right) x^{-3/2} = \frac{-1}{4} x^{-3/2}$

$h^{(3)}(x) = \frac{-1}{4}\left(\frac{-3}{2}\right) x^{-5/2}$

$= \frac{3}{8} x^{-5/2}$

Ex: $f(x) = (3x+2)^3$. find $f''(x)$.

$f'(x) = 3(3x+2)^2 \cdot 3 = 9(3x+2)^2$

$f''(x) = 9(2)(3x+2)^1 \cdot 3$

$= 54(3x+2)$